Correlation of the Sudden Freezing Point in Nonequilibrium Nozzle Flows

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Nomenclature

h = enthalpy
H_0 = stagnation enthalpy
k_r = recombination rate constant
M_a = molecular weight of atomic species
r_t = throat radius
R = gas constant for the undissociated species
T = temperature
U = velocity
x = distance along nozzle
α = degree of dissociation
θ = nozzle half angle
ρ = density

Subscripts

e = equilibrium
f = freeze conditions
0 = stagnation
r = reference conditions ($T_r = 273.16$°K)
t = throat condition

THE flow of air in chemical nonequilibrium through a wind tunnel nozzle or the products of combustion through a propulsion system nozzle is an extremely complicated process, a process which, at present, can only be treated analytically by elaborate machine calculations. It is important then to investigate any simplified methods of analysis or correlation of such flows in order to obtain the most benefit from existing solutions or data. Bray[1] has proposed one such simplified analysis in which the flow is assumed to be in chemical equilibrium down to a certain point, termed the "sudden freezing point"; beyond this point the flow is assumed to be frozen with constant composition. Bray[2] has also pointed out that for a given nozzle geometry, the static enthalpy at the "sudden freezing point" correlates well with the upstream stagnation entropy of the flow. The purpose of the present note is to point out that the static enthalpy at the "sudden freezing point" also correlates well with another parameter that includes the nozzle size and that arises as a similarity parameter in the species continuity equation.

Consider the simple problem of steady one-dimensional flow of a reacting diatomic gas. The gas is made up of atoms and molecules, and recombination is a three-body process whereas dissociation is a two-body process. The dissociation and recombination of oxygen, for example, occurs by such a process.

The species continuity equation for such a flow is

$$\frac{d\alpha}{dx} = -\frac{k_r\rho^2}{2M_A{}^2u}\,\alpha^2(1-\alpha)\left\{1 - \frac{\alpha_e{}^2}{\alpha^2}\,\frac{(1-\alpha)}{(1-\alpha_e)}\right\} \tag{1}$$

where α is the degree of dissociation. The recombination rate constant k_r is a function of temperature, and it is necessary to specify this temperature dependence to obtain a proper similarity parameter. The present discussion is limited, for simplicity, to the consideration of oxygen so that the recombination rate constant is taken as $k_{r_0}\,(3000/T)^2$, which is consistent with the work of Ref. 3. In this case

Received January 26, 1965.

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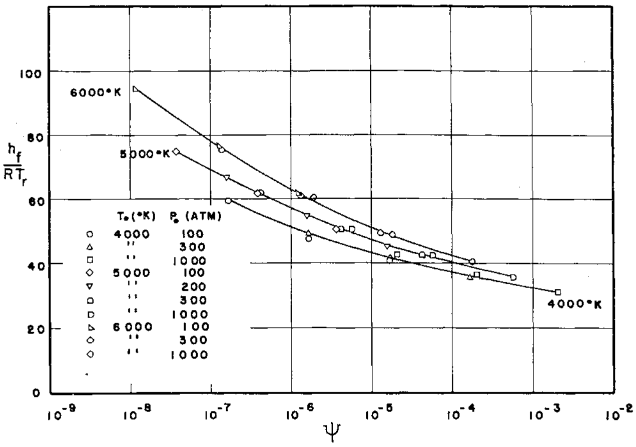

Fig. 1 Correlation of freeze enthalpy.

Eq. (1) is written

$$\frac{d\alpha}{dx} = -k_{r_0}\frac{(3000)^2}{T^2}\,\frac{\rho^2}{2M_A{}^2u}\,\alpha^2(1-\alpha)\times\left\{1 - \frac{\alpha_e{}^2}{\alpha^2}\,\frac{(1-\alpha)}{(1-\alpha_e)}\right\} \tag{2}$$

The species continuity equation is now nondimensionalized by introducing

$$\rho^* = \rho/\rho_t \qquad u^* = u/(2H_0)^{1/2} \qquad x^* = x\theta/r_t \qquad T^* = RT/2H_0$$

The nondimensional species continuity equation is

$$\frac{d\alpha}{dx^*} = -\left\{\frac{k_{r_0}}{8\ 2^{1/2}}\,\frac{\rho_t{}^2R^2r_t 9\times 10^6}{M_A{}^2H_0{}^{5/2}\theta}\right\}\frac{\rho^{*2}}{T^*u^*}\,\alpha^2\times(1-\alpha)\left\{1 - \frac{\alpha_e{}^2}{\alpha^2}\,\frac{(1-\alpha)}{(1-\alpha_e)}\right\} \tag{3}$$

It would appear that the parameter $\psi = k_{r_0}\rho_t{}^2r_tR^2/M_A{}^2H_0{}^{5/2}\theta$ determines, to a large extent, the rate of production (or loss) of atoms and therefore should control the "sudden freezing point." That this is true is seen in Fig. 1 where the "sudden freezing point" taken from Ref. (3) is plotted as a function of the preceding nondimensional parameter. The sudden freezing enthalpy is dependent to a large extent upon this parameter and to a lesser degree on the stagnation temperature.

The nondimensional parameter ψ is roughly the Damköhler number or ratio of the mechanical time to chemical time in the nozzle and as such is a logical parameter for correlating the conditions at the "sudden freezing point."

The choice of nondimensionalizing the density by the throat density was made for convenience since in many nozzles the flow is in equilibrium at least down to the throat. It may well be, that the throat density could be replaced by the upstream stagnation density with similar results. It is also possible that other characteristics of the flow might correlate well with the forementioned parameter.

References

[1] Bray, K. N. C., "Atomic recombination in a hypersonic wind tunnel nozzle," J. Fluid Mech. 6, 1–32 (July 1959).

[2] Bray, K. N. C., "Simplified sudden-freezing analysis for nonequilibrium nozzle flows," ARS J. 31, 831–834 (1961).

[3] Boyer, D. W., Eschenroeder, A. Q., and Russo, A. L., "Approximate solutions for non-equilibrium air flow in hypersonic nozzles," Cornell Aeronautical Lab. Rept. No. AD-1345-W-3; also Arnold Engineering Development Center TN-60-181 (August 1960).